

# Laboratory Lectures: Electrothermal Processing

## - Experience number 2 -

*„Induction Melting: investigation on the behavior of  
induction crucible furnaces“*

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## 1. Theoretical Fundamentals

As already anticipated in the first laboratory experience, electromagnetic induction occurs only when a variable (time-dependent) current is supplied to the primary conductor (inductor). Let us take into account an alternating current  $I_1$  flowing into a cylindrical coil at the frequency  $f$  (Fig. 1): an alternating magnetic field, at the same frequency, is generated by  $I_1$ . Defining as  $\mu_r$  the relative magnetic permeability of a cylindrical billet, the magnetic field penetrates within the billet according to penetration depth

$$\delta = \frac{1}{\sqrt{\pi \cdot f \cdot \mu_r \mu_0 \cdot k}} \quad [m] \quad (1)$$

where  $k \left[\frac{S}{m}\right]$  is the electrical conductivity of the billet workpiece.

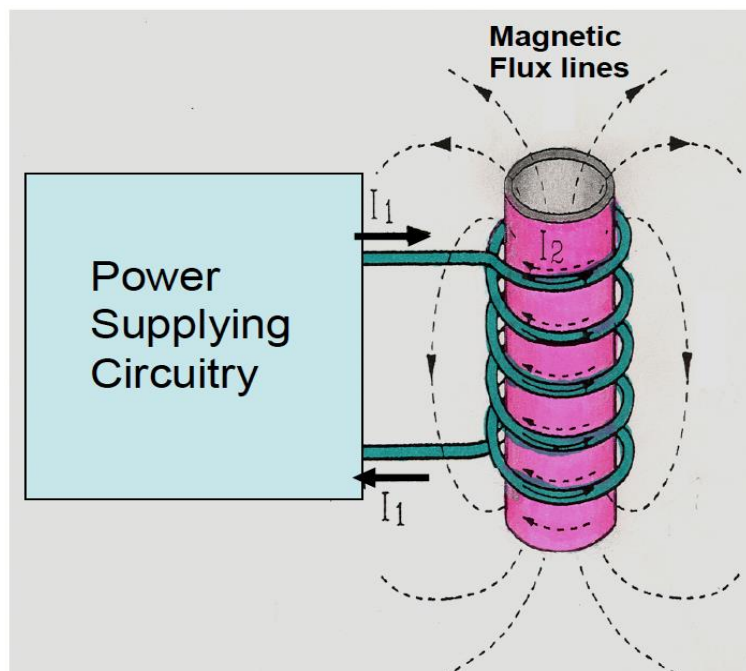


Figure 1: Principle of electromagnetic induction

Furthermore, the alternating magnetic field induces a second current  $I_2$  inside the workpiece, which produces Joule Losses and heats the billet up. Current  $I_2$  is in opposite phase to the primary one ( $I_1$ ), to reduce the total magnetic field and the global magnetic energy in the setup. In the hypothesis of the cylindrical billet to be made of a conductive material with the wall thickness  $\delta \ll d_2$  ( $d_2 =$  diameter of the billet), the entire induction heating system can be modeled as a transformer, in which the coil represents the primary, while the cylindrical billet is the secondary. The gap between the primary and the billet represents the core of the transformer. Therefore, transformer's equations can be applied:

$$\underline{U} = \underline{I}_1 * (\underline{R}_1 + j\omega L_1) + \underline{I}_2 j\omega M \quad (2)$$

$$\underline{0} = \underline{I}_1 j\omega M + \underline{I}_2 (\underline{R}_2 + j\omega L_2) \quad (3)$$

A transformation ratio between the two currents  $I_1/I_2$  can be defined:

$$p = \frac{-j\omega M}{\underline{R}_2 + j\omega L_2} \quad (4)$$

where  $p$  is a complex number; it assumes a real value only when  $R_2 = 0$  (superconducting workpiece either a short-circuited secondary coil with null resistance). Previous assumptions are allowed in the case of ideal transformers including closed iron core; they cannot be applied in our case, since the resistance of the workpiece assumes a non-negligible value. The more  $R_2$  rises, the smaller  $p$  becomes. In the previous real case, mutual inductance  $M$  can furthermore be defined as:

$$M_{12} = \frac{\Psi_{12}}{\underline{I}_1} [H] \quad (5)$$

where  $\Psi_{12} = n_2 \cdot \Phi_{12} [Wb]$  represents the total magnetic flux concatenating workpiece number 2, due to current  $I_1$ . Mutual inductance therefore quantifies the influence of primary coil's current on the secondary, when the primary magnetic field crosses the secondary workpiece.

According to the previous assumptions, it is possible to assume the secondary workpiece as a short-circuited conductor, with thickness equal to  $\delta$  and  $n_2$  density of turns. Due to magnetic flux  $\Psi_{12}$ , induced voltage at the "terminals" of the secondary becomes

$$\underline{e}_2 = -\frac{d\Psi_{12}}{dt} = -\frac{d\underline{I}_1}{dt} \cdot M_{12} [V] \quad (6)$$

The same formulas can be applied to the primary coil, by referring to the secondary:

$$M_{21} = \frac{\Psi_{21}}{\underline{I}_2} [H] \quad (7)$$

in which

$$\Psi_{21} = n_1 * \phi_{21} [Wb] \quad (8)$$

and the induced voltage becomes

$$\underline{e}_1 = -\frac{d\Psi_{21}}{dt} = -\frac{d\underline{I}_2}{dt} * M_{21} [V] \quad (9)$$

### 1.1. Ideal (no-losses and zero-resistance) model

Let us now assume the ideal case of a perfect power transfer from the primary coil to the cylindrical billet: in this case, the induction heating system can be modeled as a transformer including a closed iron core with no losses; secondary then has null resistance  $R_2 = 0$ . Magnetic flux  $\phi_1$ , concatenating the primary coil, is now equal to the flux concatenating the secondary

$$\phi_1 = \phi_{12}$$

Therefore, equation (5) can be written as:

$$n_1 * \phi_1 = \Psi_1 = L_1 * \underline{I_1} \quad \text{and} \quad n_2 * \phi_{12} = \Psi_{12} = M_{12} * \underline{I_1}$$

And from the point of view of the secondary

$$\frac{L_1}{n_1} = \frac{M_{12}}{n_2}$$

If the last equations are multiplied between themselves, and by taking into account the equivalence  $M_{12} = M = M_{21}$  between the mutual inductances, the following result can be obtained:

$$M = \sqrt{L_1 \cdot L_2} \quad [H] \quad (10)$$

### 1.2. Real model and vector diagram

In a real induction heating system, the secondary workpiece always has a finite electrical resistance,  $R_2 \neq 0$ . Furthermore, the air gap between the primary coil and the cylindrical workpiece produces losses in the generated magnetic flux, and it can therefore not be modeled as a no-losses closed iron core anymore. Any real induction system is a transformer made of an air core. The primary magnetic flux  $\Phi_1$  becomes the sum of a useful flux, concatenating the secondary, and leakage flux  $\Phi_{1\sigma}$  which represents the losses in the magnetic flux within the air gap between the primary and the secondary:

$$\Phi_1 = \Phi_{12} + \Phi_{1\sigma} \neq \Phi_{12}$$

Mutual inductance then becomes smaller

$$\frac{M}{\sqrt{L_1 \cdot L_2}} = k < 1 \quad (11)$$

Where  $k$  represents the *COUPLING FACTOR* between the primary and the secondary. Coupling factor  $k$  is then related to the *LEAKAGE FACTOR*  $\sigma$  by the following:

$$\sigma = 1 - k^2 \tag{12}$$

The smaller the coupling factor, the higher the leakage factor; again, as the leakage factor increases, mutual inductance  $M$  becomes smaller, and the transformation ratio  $p$  lowers as well.

Assuming the induction crucible furnace to be modeled as a transformer, its *VECTOR DIAGRAM* can then be represented (Fig. 2)

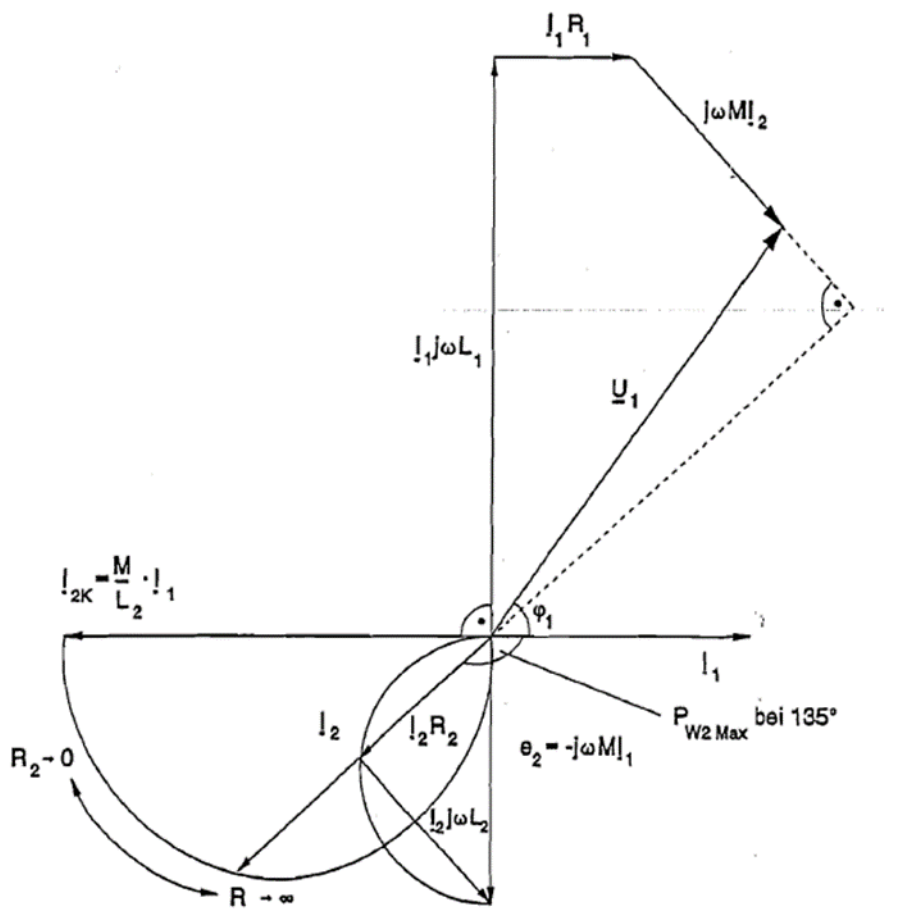


Figure 2: vector diagram of an induction crucible furnace

In Fig. 2, maximum active power  $P_{W2} = U_{R2} \cdot I_2$  can be easily recognized; phase shift between currents  $I_1$  and  $I_2$  is equal to  $135^\circ$ . This specific operating case occurs when

$$R_2 = \omega L_2$$

Where

$$R_2 = f(\text{geometry, frequency, material})$$

$$\omega L_2 = f(\text{geometry, frequency})$$

If

$$|\underline{p}^2| = \underline{p} \cdot \underline{p}^* = \frac{\omega^2 M^2}{R_2^2 + \omega^2 L_2^2} \quad (13)$$

(where  $\underline{p}^2$  becomes a real number)

It is then possible to achieve:

$$\underline{U}_1 = [R_1 + \underline{p}^2 R_2 + j\omega(L_1 - \underline{p}^2 * L_2)] \cdot \underline{I}_1 \quad (14)$$

$$\underline{U}_1 = \underline{I}_1 \cdot (R_{total} + j\omega L_{total}) \quad (15)$$

In classical transformers with very low losses (very low leakage flux), leakage factor  $\sigma$  is large enough to provide a high inductive effect of the device itself; as a direct consequence, secondary current  $I_2$  shifts close to  $I_{2K}$  (Fig. 2). In this way, current  $|I_2|$  becomes relatively independent from the workpiece material, and transferred active power to the workpiece is proportional to the AC resistance of the imaginary hollow cylinder with the thickness  $\delta$  carrying the induced current  $I_2$

$$P_w = < |I_2|^2 \cdot R_{2\sim} = |I_2|^2 \cdot \frac{\pi(d_2 - \delta)}{l \cdot \delta} \cdot \rho \quad (16)$$

where  $l$  = cylinder's length.

If

$$\delta = \frac{1}{\sqrt{\pi \cdot f \cdot \mu \cdot k}}, \quad k = \frac{1}{\rho}, \quad d_2 \gg \delta, \quad \text{and} \quad \underline{I}_2 \approx \text{constant}$$

Then it is possible to achieve

$$P_w \sim \sqrt{\rho \mu_r} \quad (17)$$

in the hypothesis of no change in the dimensions of the various inserts and with the same frequency and supplied voltage. On the other hand, when frequency and voltage change, induced current  $I_2$  becomes:

$$\underline{I}_2 \approx \frac{U_2}{\omega L_\sigma} \quad (18)$$

Due to the leakage inductance

$$L_{\sigma} \gg \frac{R_{2\sim}}{\omega} \quad (19)$$

it finally turns out that

$$P_w = |I_2|^2 \cdot R_{2\sim} \sim U_2^2 \sqrt{\frac{\rho \cdot \mu_r}{f^3}} \quad (20)$$



## 2. Experimental Setup

The following picture (Fig. 3) gives a schematic representation of the induction melting setup involved in this experimental activity: a cylindrical metal workpiece is inductively heated by a cylindrical copper inductor, specifically designed for induction melting; the total amount of Joule Losses generated within the workpiece during time interval  $\Delta t$  is equal to:

$$Q_w = R_w \cdot J^2 \cdot \Delta t \quad [J] \quad (21)$$

where  $R_w [\Omega m]$  is the equivalent electrical resistance of the workpiece with wall thickness  $\delta$  and  $J [\frac{A}{m^2}]$  is the induced current density within the workpiece.

Each hollow workpiece is water cooled through an internal cooling circuit: in this way, thermal losses generated within the workpiece by Joule effect are exported and temperature of the probe is kept constant at 25 [°C]. Therefore, it is possible to both calculate the power transferred to the workpiece and keep safe conditions during the investigation. In a more general way, thermal energy losses within a certain body are proportional to its mass,  $m [kg]$ , heat capacity at constant pressure  $c_p [\frac{J}{kgK}]$  and the change in temperature  $\Delta\vartheta [°C]$  of the investigated body:

$$Q_{,thermal} = c_p \cdot m \cdot \Delta\vartheta \quad [J] \quad (22)$$

since mass is equal to the product between the material's density and its volume,  $m = \rho \cdot V [kg]$ , formula (22) becomes

$$Q_{,thermal} = c_p \cdot \rho \cdot V \cdot \Delta\vartheta \quad [J] \quad (23)$$

Total thermal power transferred to the metal workpiece in the interval of time  $\Delta t$  is equal to

$$P_{,thermal} = \frac{\Delta Q_{,thermal}}{\Delta t} = \frac{c_p \cdot \rho \cdot V \cdot \Delta\vartheta}{\Delta t} \quad [W] \quad (24)$$

Referring to the water-cooling system linked to the probe, term  $V/\Delta t [\frac{kg}{s}]$  represents the flow of water necessary to export Joule Losses  $\Delta Q_{,thermal}$  from the workpiece and keep it at the targeted ambient temperature during the interval of time  $\Delta t$ . It is finally possible to rewrite formula (24) in the following way:

$$P_{thermal} = c_p \cdot \rho \cdot \dot{V} \cdot (\vartheta_{out} - \vartheta_{in}) [W] \quad (25)$$

Where  $\dot{V} [\frac{m^3}{s}]$  is the water flow through the probe circuit and  $\vartheta_{out} - \vartheta_{in} [^{\circ}C]$  represents the variation of temperature between the ingoing water to the workpiece and its outlet.

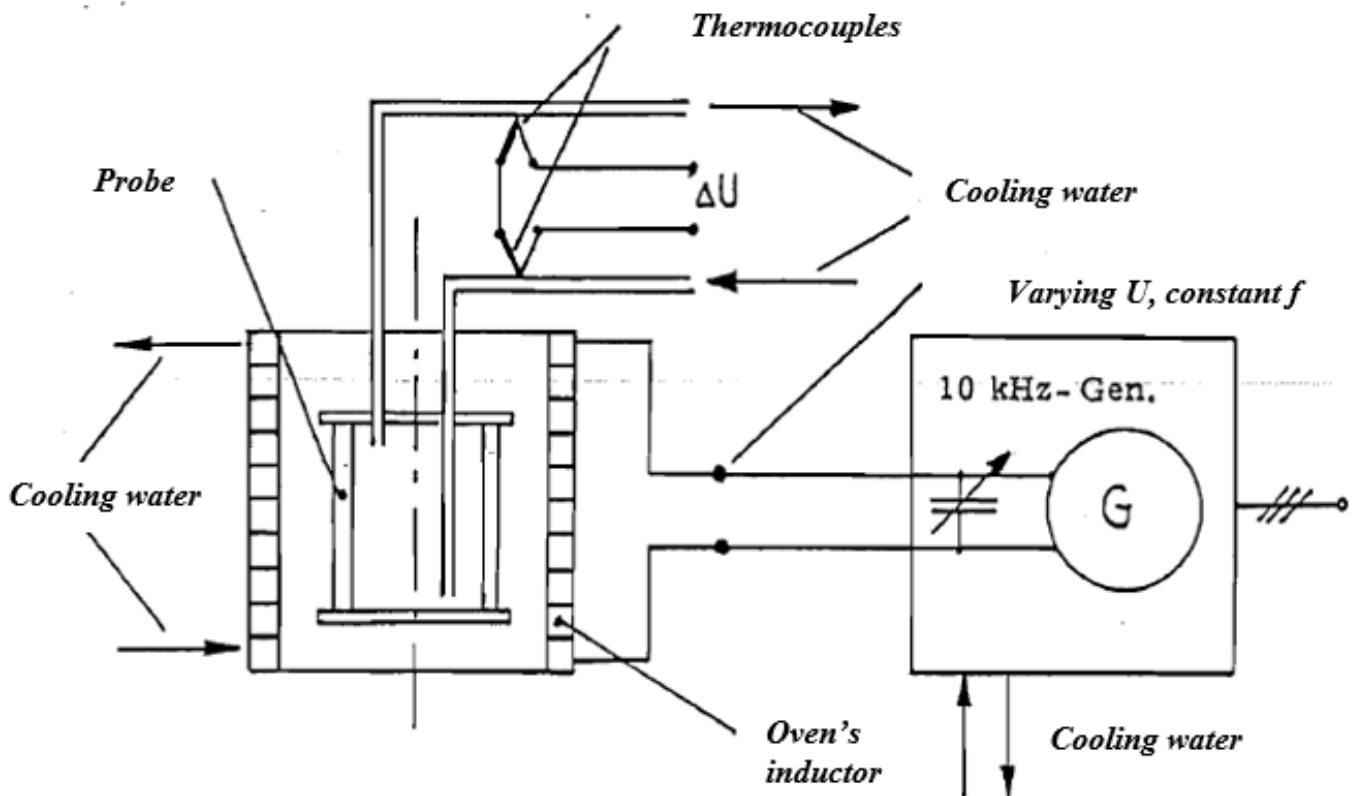


Figure 3: scheme of the experimental arrangement

## Introduction

The experimental setup consists of a DEMAG induction crucible furnace (including pre-installed sensors), the EMA-TEC converter, a Dolphin Expert Key 100L data-collecting board and its relative software, necessary for the analysis of the data. EMA-TEC converter supplies the required electrical power to the induction furnace, while the Dolphin Expert Key 100L box works as an interface between the converter, the sensors and the computer. ProfiSignal Klicks is the working software which displays and analyses the data from the experimental setup: from the same software, it is also possible to directly control all the electrical parameters of the converter. A schematic view of the communicating network between each component of the experimental setup is shown in Fig. 4.

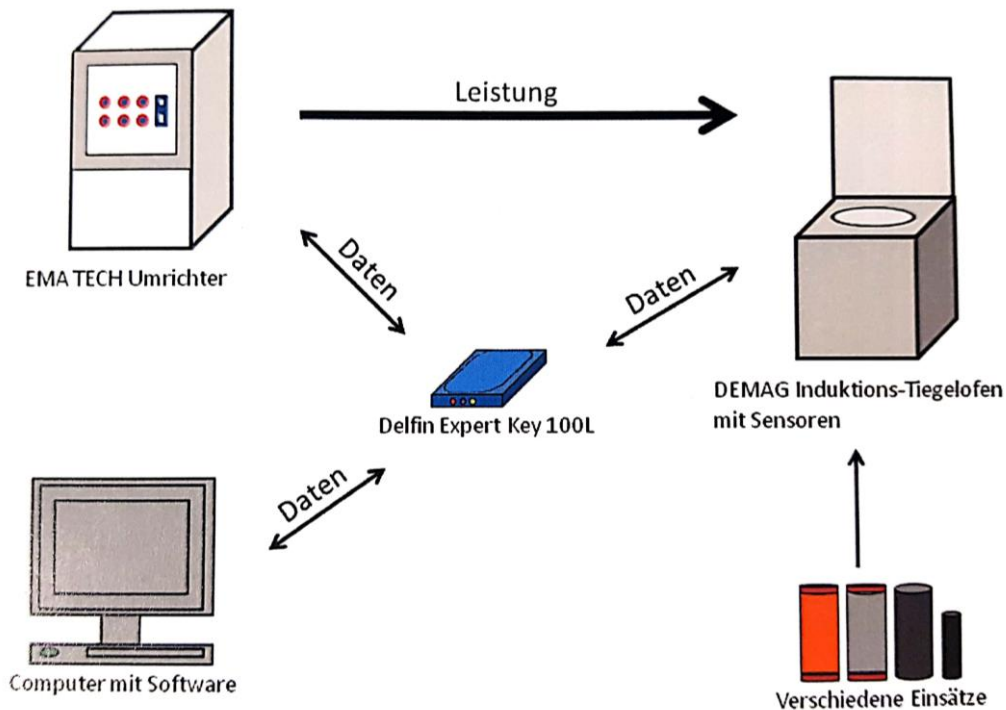


Figure 4: communicating network of the experimental setup

## DEMAG Induction Crucible Furnace and Sensors

Fig. 5 shows the full induction furnace installation adopted for the experimental activity: the crucible is surrounded by a cylindrical, water-cooled copper inductor (Fig. 5.1), including sensors and thermocouples, and a capacitor bank, necessary for the matching of frequency. Water-cooling is supplied by a set of three pumps (copper tubes above the furnace), each one of them connected to a different element of the setup: pump number 3 (right) is directly linked to the inductor of the furnace; pumps 1 (left side) and 2 (center) are connected to the heated probe inside the crucible. The superior cover of the crucible is made of refractory material to protect users for the heat generated by the same oven; the furnace can be tilted thanks to the crank on the right side, therefore locked on a fixed position by a rod.

Active power transferred to the workpiece is directly measured by a set of 4 cylindrical probes (Fig. 5.2) made of copper, aluminium, brass and steel. Each probe is directly water-cooled by pumps 1 and 2, previously described. For the last part of the experiment, a hollow steel cylinder is used.

Sensors used in the experimental setup include: three water-flow sensors, one thermocouple and a Rogowski probe. Flow sensors not only measure the water flow of the cooling water, but also water temperature. Thermocouple is adopted to measure the temperature evolution of the steel cylinder, while the Rogowski coil measures the supplied current to the inductor by the oscillating circuit.



*Figure 5: experimental setup. The induction crucible furnace is shown below, including the metal probe and the water-cooling system. On the left side, the Delphin Expert Key 100L, collecting all of the data coming from the setup itself.*





Figure 5.1: detailed view of the induction crucible furnace. Copper inductor and capacitors bank



Figure 5.2: set of the four metal probes, plus the steel cylinder



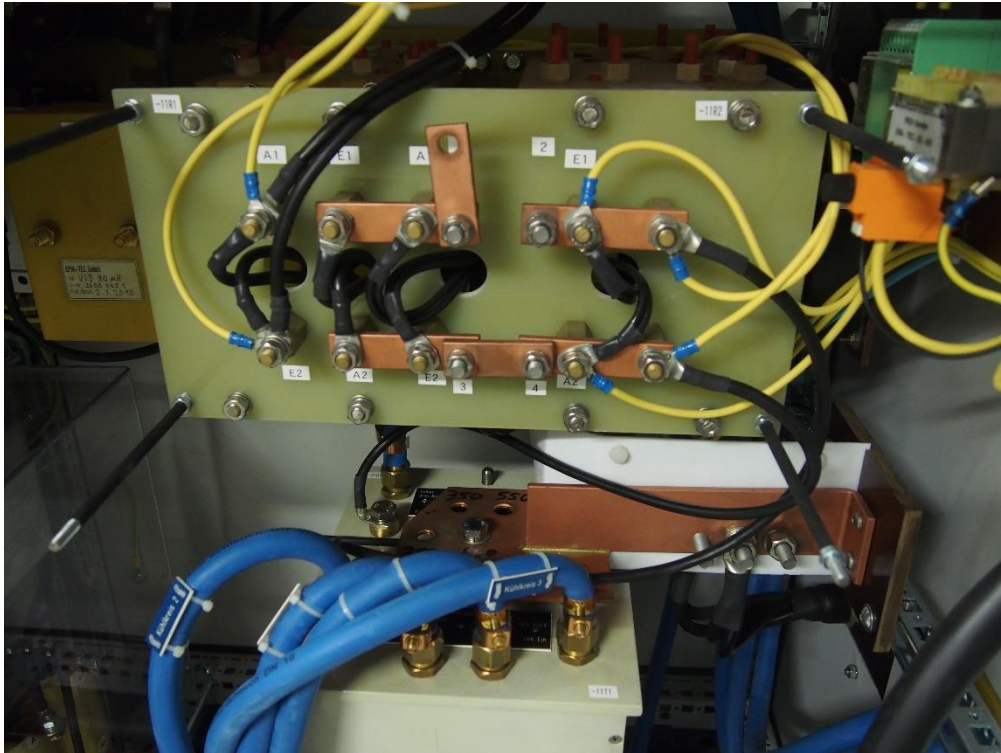
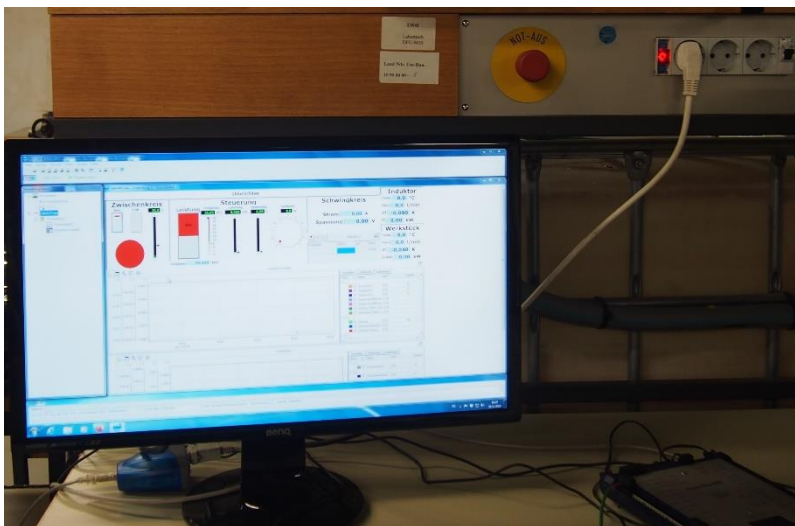


Figure 6.1: output transformer installed inside the generator

## ProfiSignal Klicks software



ProfiSignal software allows the user to deal with the entire set of electrical and thermal parameters involved in this experiment; the collection of all data is provided by Delphin board and then transferred to the software. ProfiSignal interface not only shows parameters values, but also allows the user to directly control the EMA-TEC generator, especially its output power and voltage. In summary, the interface of ProfiSignal software includes: the DC link voltage value, a central window representing the main outputs of the inverter (frequency, voltage and power) and a right-side window with the thermal and electrical parameters of both the inductor and the workpiece. Flow value of the water circulating in the cooling circuit is also shown.



### 3. Tasks

- 1) Samples (probes) of copper, aluminium, brass and iron must be thermally investigated and compared in terms of absorbed power: evaluate the absorbed power directly from the the PorfiSignal software. What is the material which absorbs the maximum power? Give an explanation to this behavior.
- 2) Calculate the efficiency for each sample and compare the efficiency between all of the probes.
- 3) A hollow steel cylinder is heated up by supplying constant voltage  $U \approx 150 [V]$  at the generator. Give the graph representing the evolution of temperature of the cylinder vs time; record the temperature with the help of a recorder. Give your comments to the curve.
- 4) Cylinder form 3) must be heated up to its final temperature ( $\approx 800 [^{\circ}C]$ ) by supplying constant power  $P \approx 12 [kW]$  from the generator.

### Questions

- Why is there a pronounced kink in the temperature curve of task 3)?
- In task 4), why does not a regular temperature-time function occur, despite constant power is supplied to the steel cylinder by the generator?
- What happens during the cool-down?